Homework V

Due: Dec. 4. (Fri) 23:59 PM

I. REMARK

- Reading materials: Ch 5.5, 5.6, 6.1-6.8 in the textbook.
- Don't write just an answer. Please describe enough processes to justify your answer (Korean is also OK!!).
- Check the due date!!!
- Better the last smile than the first laughter.

II. PROBLEM SET

1) Let A be a real 2×2 matrix with a complex eigenvalue $\lambda = a - bi$ ($b \neq 0$) and an associated eigenvector $\mathbf{v}$ in $\mathbb{C}^2$.
   a) Show that A(Re $\mathbf{v}$) = a(Re $\mathbf{v}$) + b(Im $\mathbf{v}$) and A(Im $\mathbf{v}$) = $-b$(Re $\mathbf{v}$) + a(Im $\mathbf{v}$).
   b) Verify that if P and C are given as in Theorem 9 in Ch. 5.5, then $AP = PC$.

2) Classify the origin as an attractor, repeller, or saddle point of the dynamical system $\mathbf{x}_{k+1} = A\mathbf{x}_k$. A is given as
$$A = \begin{bmatrix} 0.3 & 0.4 \\ -0.3 & 1.1 \end{bmatrix}$$

3) Let $\mathbf{y}$= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{u}$ = $\begin{bmatrix} 4 \\ -7 \end{bmatrix}$. Write $\mathbf{y}$ as the sum of two orthogonal vectors, one in Span $\{\mathbf{u}\}$ and one orthonal to $\mathbf{u}$.

4) Find the best approximation to $\mathbf{z}$ by vectors of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$. Also, find the distance from $\mathbf{z}$ to the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.
$$\mathbf{z} = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix} \mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix} \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

5) Let W be a subsapce of $\mathbb{R}^n$. Show that dim W + dim $W^{\perp}$ = n.

6) Find an orthogonal basis for the column space of the matrix (Use the Gram-Schmidt process).
$$\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$$

7) Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$.
$$A = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

8) Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the given data points. $\{(0,1),(1,1),(2,2),(3,2)\}$.

9) For x and y in $\mathbb{P}_3$, define $< x, y >= x(-3)y(-3) + x(-1)y(-1) + x(1)y(1) + x(3)y(3)$. Let $p_0(t) = 1$, $p_1(t) = t$, and $p_2(t) = t^2$.
   a) Compute the orthogonal projection of p_2 onto the subspace spanned by p_0 and p_1.
   b) Find a polynomial q that is orthogonal to p_0 and p_1, such tat $\{p_0, p_1, p_2\}$ is an orthogonal basis for Span $\{p_0, p_1, p_2\}$. Scale the polynomial q so that its vector of values at $(-3, -1, 1, 3)$ is $(1, -1, -1, 1)$.