Homework IV

Due: Nov. 20. (Fri) 23:59 PM

I. Remark

- Reading materials: Ch 4.1-4.7, 5.1-5.4 in the textbook.
- Don't write just an answer. Please describe enough processes to justify your answer (Korean is also OK!!).
- Check the due date!!!
- Better the last smile than the first laughter.

II. Problem Set

1) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices, and define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$, where
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
   a) Show that T is a linear transformation.
   b) Show that the range of T is the set of B in $M_{2\times 2}$ with the property that $B^T = B$.
   c) Describe the kernel of T.

2) Find a basis for the set of vectors in $\mathbb{R}^3$ in the plane $x_1 + 2x_2 + x_3 = 0$.

3) Let $\mathbb{P}_2$ be the set of polynomials where the degree of polynomials is 2.
   a) The set $B = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the change-of-coordinates matrix from the basis B to the standard basis $C = \{1, t, t^2\}$.
   b) Find the coordinate vector of $\mathbf{p}(t) = 3 + t - 6t^2$ relative to B.

4) Show that the set of polynomials $\{1, 1 - t, 2 - 4t + t^2, 6 - 18t + 9t^2 - t^3\}$ is a basis of $\mathbb{P}_3$.

5) Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} is an eigenvalue of A^{-1}.

6) Diagonalize the matrix if possible.
$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

7) Assume the mapping $T : \mathbb{P}_2 \to \mathbb{P}_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$ is linear. Find the matrix representation of T relative to the basis $B = \{1, t, t^2\}$.
8) Let $T : \mathbb{P}_2 \to \mathbb{P}_4$ be the transformation that maps a polynomial $\mathbf{p}(t)$ into the polynomial $\mathbf{p}(t) + t^2\mathbf{p}(t)$.
   a) Find the image of $\mathbf{p}(t) = 2 - t + t^2$.
   b) Show that T is a linear transformation.
   c) Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3, t^4\}$.