Homework I

Due: Sep. 25. (Fri) 23:59 PM

I. REMARK

- Reading materials: Ch 1.1-1.7 in the textbook.
- Don't write just an answer. Please describe enough processes to justify your answer (Korean is also OK!!).
- "unique solution" = "only one solution"

II. PROBLEM SET

1) Solve the system using an augmented matrix.

$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

2) Find the general solution of the systems whose augmented matrix is given in

$$\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$$

3) Determine if the system is consistent.

$$\begin{aligned} x_1 - 2x_4 &= -3 \\ 2x_2 + 2x_3 &= 0 \\ x_3 + 3x_4 &= 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 &= 5 \end{aligned}$$

4) Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solution. Give separate answers for each part.

$$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$$

5) Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

6) A system of linear equations with fewer equations than unknowns (variables) is called an underdetermined system. Suppose that the system is consistent. Explain why there must be an infinite number of solutions.

7) Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

8) Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

9) For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$$

10) Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 3x_1 + x_2 - 5x_3 &= 9 \\ x_2 + 4x_3 &= 0 \end{aligned}$$

11) Do the columns of B span $\mathbb{R}^4$? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for every $\mathbf{y}$ in $\mathbb{R}^4$?

$$B = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

12) Suppose A is a 3×3 matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^3$ with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Explain why the columns of A must span $\mathbb{R}^3$.

13) Determine if the system has a nontrivial solution.

$$\begin{aligned} 2x_1 - 5x_2 + 8x_3 &= 0 \\ -2x_1 - 7x_2 + x_3 &= 0 \\ 4x_1 + 2x_2 + 7x_3 &= 0 \end{aligned}$$

14) Determine if the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

15) Mark each statement True of False. Justify each answer.
    a) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.
    b) The columns of any 4×5 matrix are linearly dependent.
    c) If $\mathbf{z}$ is in $\text{Span}\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.