Midterm Exam

BX25203 Engineering Mathematics (I), Fall 2020
School of BioMedical Convergence Engineering, PNU
Oct. 21. 10:30 - 12:00

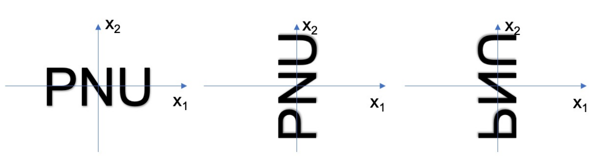
I. REMARK

- This is a closed book exam. You are permitted on one page of notes.
- There are a total of 100 points in the exam. Each problem specifies its point total.
- You must SHOW YOUR WORK to get full credit.

II. PROBLEM SET

1) [20 points] Mark each statement True or False. You don't need to justify each answer in this problem.
   a) If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is in the set spanned by the columns of A. [True/False]
   b) The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable. [True/False]
   c) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in Span $\{\mathbf{x}, \mathbf{y}\}$. [True/False]
   d) If A is a 3×2 matrix, then the transformation $\mathbf{x} \to A\mathbf{x}$ cannot be one-to-one.[True/False]
   e) If A and B are 3×3 and $B = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$, then $AB = [A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3]$. [True/False]
   f) The dimension of Col A is the number of pivot columns of A. [True/False]
   g) $\mathbb{R}^2$ is the subspace of $\mathbb{R}^4$. [True/False]
   h) If the columns of $A \in \mathbb{R}^{n\times n}$ are linearly dependent, then det$A = 0$. [True/False]
   i) If H is a p-dimensional subspace of $\mathbb{R}^n$, then a linearly independent set of p vectors in H is a basis for H.
   j) If A and B are $n \times n$, then $(A + B)(A - B) = A^2 - B^2$. [True/False]

2) [15 points] $T : \mathbb{R}^2 \to \mathbb{R}^2$ first rotates points through $\pi/2$ radian, counter-clockwise ((A) to (B)) and then reflects points through the vertical x_2-axis ((B) to (C)). Find the standard matrix of T. Also, find the matrix of T^{-1}.



(A) (B) (C)

3) [20 points] Let $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 3 & 2 & -1 \\ -4 & -8 & 0 & 0 \\ -1 & 1 & 6 & -3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 0 \\ 15 \end{bmatrix}.$$

   a) Find a solution set $\mathbf{x}$ if it is consistent.
   b) Find a basis of Col$A$.
   c) Find a basis of Nul$A$.
   d) Find an LU factorization of the matrix A. Note that $L \in \mathbb{R}^{3\times 3}$ and $U \in \mathbb{R}^{3\times 4}$.

4) [10 points] A set $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and a vector $\mathbf{a}$ are given as

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 3 \end{bmatrix}, \mathbf{a} = \begin{bmatrix} -1 \\ -3 \\ 0 \\ 0 \end{bmatrix},$$

   a) Is $\mathbf{a}$ in span B?
   b) Is a set $C = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{a}\}$ a basis of $\mathbb{R}^4$?

5) [10 points] Suppose that A is a 3×3 matrix with the property that $A\mathbf{v}_1 = \mathbf{e}_1$, $A\mathbf{v}_2 = \mathbf{e}_2$ and $A\mathbf{v}_3 = \mathbf{e}_2 + \mathbf{e}_3$ where

$$\mathbf{v}_1 = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}.$$

Find A^{-1}. ($\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ denote the standard basis vectors in $\mathbb{R}^3$)

6) [15 points] A matrix A is given as

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}.$$

   a) Show that $A + A^{-1} = 2I$.
   b) Find a determinant of A^{100}.
   c) Find A^{100}.

7) [10 points] Let $C = AB \in \mathbb{R}^{3\times 3}$ where $A \in \mathbb{R}^{3\times 2}$ and $B \in \mathbb{R}^{2\times 3}$.
   a) Show that the first column vector of C is the linear combination of the column vectors of A.
   b) Is C invertable? Justify your answer.